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THE DEGREE OF AN EDGE IN CARTESIAN PRODUCT AND COMPOSITION OF TWO FUZZY GRAPHS

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ABSTRACT

A fuzzy graph can be obtained from two given fuzzy graphs using Cartesian product and composition. In this paper degree of an edge and total degree of an edge are introduced and we find the degree of an edge in fuzzy graphs formed by these operations in terms of the degree of edges in the given fuzzy graphs in some particular cases.

KEYWORDS: Cartesian Product, Composition, Degree of a Vertex, Degree of an Edge

INTRODUCTION

Fuzzy graph theory was introduced by Azriel Rosenfeld in 1975. Though it is very young, it has been growing fast and has numerous applications in various fields. During the same time Yeh and Bang have also introduced various concepts in connectedness in fuzzy graphs. Mordeson. J. N and Peng. C. S introduced the concept of operations on fuzzy graphs. Sunitha. M. S and Vijayakumar. A discussed about the complement of the operations of union, join, Cartesian product and composition on two fuzzy graphs. The degree of a vertex in fuzzy graphs which are obtained from two given fuzzy graphs using these operations was discussed by Nagoorgani. A and Radha. K. In this paper we introduce the concept of a degree of an edge and total degree of an edge in fuzzy graphs. We study about the degree of an edge in fuzzy graphs which are obtained from two given fuzzy graphs using the operations Cartesian product and composition. In general, the degree of an edge in Cartesian product and composition of two fuzzy graphs G_1 and G_2 cannot be expressed in terms of these in G_1 and G_2. In this paper, we find the degree of an edge in Cartesian product and composition of two fuzzy graphs G_1 and G_2 in terms of the degree of edges of G_1 and G_2 in some particular cases. First we go through some basic concepts.

A fuzzy subset of a set V is a mapping σ from V to [0, 1]. A fuzzy graph G is a pair of functions G: (σ, μ) where σ is a fuzzy subset of a non-empty set V and μ is a symmetric fuzzy relation on σ, (i.e.) $\mu(x, y) \leq \sigma(x) \wedge \sigma(y)$ for all x, y $\in$ V. The underlying crisp graph of G: (σ, μ) is denoted by G*: (V, E) where $E \subseteq V\times V$. The underlying crisp graph of G: (σ, μ) is denoted by G^*: (V, E) where $E \subseteq V \times V$. Throughout this paper, G_1: (σ_1, μ_1) and G_2: (σ_2, μ_2) denote two fuzzy graphs with underlying crisp graphs G_1^*: (V_1, E_1) and G_2^*: (V_2, E_2) with $|V_i| = p_i$, i = 1, 2. Also $d_{G_i^*}(u_i)$ denotes the degree of u_i in G_i^*.

Let G: (σ, μ) be a fuzzy graph on G^*: (V, E). The degree of a vertex u is $d_G(u) = \sum_{u\neq v}\mu(u,v)$. Since $\mu(u, v) > 0$ for uv $\in$ E, $\mu(u, v) = 0$ for uv $\notin$ E. This is equivalent to $d_G(u) = \sum_{uv\in V}\mu(u,v)$. The minimum degree of G is $\delta(G) = \wedge\{d_G(v), \forall v \in V\}$ and the maximum degree of G is $\Delta(G) = \vee\{d_G(v), \forall v \in V\}$.

Let G: (σ, μ) be a fuzzy graph on G^*: (V, E). The total degree of a vertex u∈ V is defined by $td_G(u) = \sum_{u\neq v}\mu(u,v) + \sigma(u)$. Since μ(u, v) > 0 for uv ∈ E, μ(u, v) = 0 for uv ∉ E. This is equivalent to $td_G(u) = d_G(u) + \sigma(u)$.

The order and size of a fuzzy graph G are defined by $O(G) = \sum_{u\in V}\sigma(u)$ and $S(G) = \sum_{uv\in E}\mu(uv)$.

Let G^*: (V, E) be a graph and let e = uv be an edge in G^*. Then the degree of an edge e = uv ∈ E is defined by $d_{G^*}(uv) = d_{G^*}(u) + d_{G^*}(v) - 2$.

Definition [4]: Let $G^* = G_1^* \times G_2^* = (V, E)$ be the Cartesian product of two graphs G_1^* and G_2^*, where $V = V_1 \times V_2$ and $E = \{(u,u_2)(u,v_2) : u \in V_1, u_2v_2 \in E_2\} \cup \{(u_1,w)(v_1,w) : u_1v_1 \in E_1, w \in V_2\}.$ Then the Cartesian product of two fuzzy graphs G_1 and G_2 is a fuzzy graph $G = G_1 \times G_2 : (\sigma_1 \times \sigma_2, \mu_1 \times \mu_2)$ defined by

$$(\sigma_1 \times \sigma_2)(u_1,u_2) = \sigma_1(u_1) \wedge \sigma_2(u_2), \forall (u_1,u_2) \in V \text{ and}$$

$$(\mu_1 \times \mu_2)((u,u_2)(u,v_2)) = \sigma_1(u) \wedge \mu_2(u_2v_2), \forall u \in V_1, \forall u_2v_2 \in E_2,$$

$$(\mu_1 \times \mu_2)((u_1,w)(v_1,w)) = \sigma_2(w) \wedge \mu_1(u_1v_1), \forall w \in V_2, \forall u_1v_1 \in E_1.$$

Definition [4]: Let $G^* = G_1^* \circ G_2^* = (V, E)$ be the Composition of two graphs G_1^* and G_2^*, where $V = V_1 \times V_2$ and $E = \{(u,u_2)(u,v_2) : u \in V_1, u_2v_2 \in E_2\} \cup \{(u_1,w)(v_1,w) : u_1v_1 \in E_1, w \in V_2\} \cup \{(u_1,u_2)(v_1,v_2) : u_1v_1 \in E_1, u_2 \neq v_2\}.$ Then the Composition of two fuzzy graphs G_1 and G_2 is a fuzzy graph $G = G_1[G_2] = G_1 \circ G_2 : (\sigma_1 \circ \sigma_2, \mu_1 \circ \mu_2)$ defined by $(\sigma_1 \circ \sigma_2)(u_1,u_2) = \sigma_1(u_1) \wedge \sigma_2(u_2), \forall (u_1,u_2) \in V$ and

$$(\mu_1 \circ \mu_2)((u,u_2)(u,v_2)) = \sigma_1(u) \wedge \mu_2(u_2v_2), \forall u \in V_1, \forall u_2v_2 \in E_2,$$

$$(\mu_1 \circ \mu_2)((u_1,w)(v_1,w)) = \sigma_2(w) \wedge \mu_1(u_1v_1), \forall w \in V_2, \forall u_1v_1 \in E_1,$$

$$(\mu_1 \circ \mu_2)((u_1,u_2)(v_1,v_2)) = \sigma_2(u_2) \wedge \sigma_2(v_2) \wedge \mu_1(u_1v_1), \forall u_2 \neq v_2, \forall u_1v_1 \in E_1.$$

Theorem* [2]: If $G_1 : (\sigma_1, \mu_1)$ and $G_2 : (\sigma_2, \mu_2)$ are two fuzzy graphs such that $\sigma_1 \leq \mu_2$, then $\sigma_2 \geq \mu_1$ and vice versa.

EDGE DEGREE AND TOTAL EDGE DEGREE OF A FUZZY GRAPH

Edge Degree of a Fuzzy Graph

Let G: (σ, μ) be a fuzzy graph on G^*: (V, E).

The degree of an edge uv is $d_G(u, v) = d_G(u) + d_G(v) - 2\mu(u, v)$. Since μ(u, v) > 0 for uv∈ E, μ(u, v) = 0 for uv∉ E.

This is equivalent to $d_G(uv) = \sum_{\substack{uw\in E \\ w\neq v}}\mu(uw) + \sum_{\substack{wv\in E \\ w\neq u}}\mu(wv)$.

The minimum degree and maximum degree of G are $\delta_E(G) = \wedge\{d_G(uv), \forall uv\in E\}$ and $\Delta_E(G) = \vee\{d_G(uv), \forall uv\in E\}$.

Total Edge Degree of a Fuzzy Graph

Let G: (σ, μ) be a fuzzy graph on G^*: (V, E). The total degree of a vertex u∈ V is defined by $td_G(uv) = d_G(u) + d_G(v) - \mu(uv)$. Since μ(uv) > 0 for uv∈ E, μ(uv) = 0 for uv∉ E.

This is equivalent to $td_G(uv) = \sum_{\substack{uw\in E \\ w\neq v}} \mu(uw) + \sum_{\substack{wv\in E \\ w\neq u}} \mu(wv) + \mu(uv) = d_G(uv) + \mu(uv)$.

Example

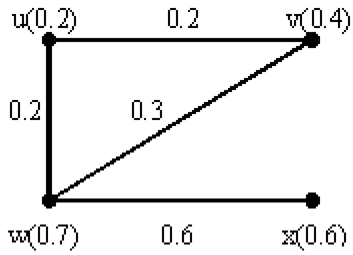

Figure 1: Fuzzy Graph G: (σ, μ)

$d_G(u) = 0.4$, $td_G(u) = 0.6$, $\delta(G) = 0.4 = d_G(u)$ and $\Delta(G) = 1.1 = d_G(w)$.

$d_G(uv) = 0.5$, $td_G(uv) = 0.7$, $\delta_E(G) = 0.5 = d_G(uv) = d_G(wx)$ and $\Delta_E(G) = 1.1 = d_G(uw)$.

DEGREE OF AN EDGE IN CARTESIAN PRODUCT

By definition, for any $(u_1,u_2)\in V_1\times V_2$ and $((u_1,u_2)(v_1,v_2))\in E$ with $u_1 = v_1, u_2\neq v_2$ or $u_1\neq v_1, u_2 = v_2$.

$$d_{G_1\times G_2}((u_1,u_2)(v_1,v_2)) = \sum_{\substack{(u_1,u_2)(w_1,w_2)\in E, \\ (w_1,w_2)\neq(v_1,v_2)}} (\mu_1\times\mu_2)((u_1,u_2)(w_1,w_2)) + \sum_{\substack{(w_1,w_2)(v_1,v_2)\in E, \\ (w_1,w_2)\neq(u_1,u_2)}} (\mu_1\times\mu_2)((w_1,w_2)(v_1,v_2)).$$

- If $u_1 = v_1, u_2\neq v_2$, then

$$d_{G_1\times G_2}((u_1,u_2)(u_1,v_2)) = \sum_{\substack{(u_1,u_2)(w_1,w_2)\in E, \\ (w_1,w_2)\neq(u_1,v_2)}} (\mu_1\times\mu_2)((u_1,u_2)(w_1,w_2)) + \sum_{\substack{(w_1,w_2)(u_1,v_2)\in E, \\ (w_1,w_2)\neq(u_1,u_2)}} (\mu_1\times\mu_2)((w_1,w_2)(u_1,v_2)).$$

$$= \sum_{\substack{(u_1,u_2)(u_1,w_2)\in E, u_1=w_1, \\ w_2\neq v_2}} (\mu_1\times\mu_2)((u_1,u_2)(u_1,w_2)) + \sum_{(u_1,u_2)(w_1,u_2)\in E, u_2=w_2} (\mu_1\times\mu_2)((u_1,u_2)(w_1,u_2))$$

$$+ \sum_{\substack{(u_1,w_2)(u_1,v_2)\in E, w_1=u_1, \\ w_2\neq u_2}} (\mu_1\times\mu_2)((u_1,w_2)(u_1,v_2)) + \sum_{(w_1,v_2)(u_1,v_2)\in E, w_2=v_2} (\mu_1\times\mu_2)((w_1,v_2)(u_1,v_2)).$$

$$= \sum_{\substack{u_2w_2\in E_2, u_1=w_1, \\ w_2\neq v_2}} \sigma_1(u_1)\wedge\mu_2(u_2w_2) + \sum_{u_1w_1\in E_1, u_2=w_2} \mu_1(u_1w_1)\wedge\sigma_2(u_2) + \sum_{\substack{w_2v_2\in E_2, w_1=u_1, \\ w_2\neq u_2}} \sigma_1(u_1)\wedge\mu_2(w_2v_2) +$$

$$\sum_{w_1u_1\in E_1, w_2=v_2} \mu_1(w_1u_1)\wedge\sigma_2(v_2).$$

$$\therefore d_{G_1\times G_2}((u_1,u_2)(u_1,v_2)) = \sum_{w_2\in V_2, w_2\neq v_2}\sigma_1(u_1)\wedge\mu_2(u_2w_2) + \sum_{w_1\in V_1}\mu_1(u_1w_1)\wedge\sigma_2(u_2) + \sum_{w_2\in V_2, w_2\neq u_2}\sigma_1(u_1)\wedge\mu_2(w_2v_2) + \sum_{w_1\in V_1}\mu_1(w_1u_1)\wedge\sigma_2(v_2) \quad (5.1)$$

- If $u_1 \neq v_1, u_2 = v_2$, then

$$d_{G_1\times G_2}((u_1,u_2)(v_1,u_2)) = \sum_{\substack{(u_1,u_2)(w_1,w_2)\in E,\\ (w_1,w_2)\neq(v_1,u_2)}}(\mu_1\times\mu_2)((u_1,u_2)(w_1,w_2)) + \sum_{\substack{(w_1,w_2)(v_1,u_2)\in E,\\ (w_1,w_2)\neq(u_1,u_2)}}(\mu_1\times\mu_2)((w_1,w_2)(v_1,u_2)).$$

$$= \sum_{(u_1,u_2)(u_1,w_2)\in E, u_1=w_1}(\mu_1\times\mu_2)((u_1,u_2)(u_1,w_2)) + \sum_{\substack{(u_1,u_2)(w_1,u_2)\in E, u_2=w_2,\\ w_1\neq v_1}}(\mu_1\times\mu_2)((u_1,u_2)(w_1,u_2))$$

$$+ \sum_{(v_1,w_2)(v_1,u_2)\in E, w_1=v_1}(\mu_1\times\mu_2)((v_1,w_2)(v_1,u_2)) + \sum_{\substack{(w_1,u_2)(v_1,u_2)\in E, w_2=u_2,\\ w_1\neq u_1}}(\mu_1\times\mu_2)((w_1,u_2)(v_1,u_2)).$$

$$= \sum_{u_2w_2\in E_2, u_1=w_1}\sigma_1(u_1)\wedge\mu_2(u_2w_2) + \sum_{\substack{u_1w_1\in E_1, u_2=w_2,\\ w_1\neq v_1}}\mu_1(u_1w_1)\wedge\sigma_2(u_2) + \sum_{w_2u_2\in E_2, w_1=u_1}\sigma_1(v_1)\wedge\mu_2(w_2u_2)$$

$$+ \sum_{\substack{w_1v_1\in E_1, w_2=u_2,\\ w_1\neq u_1}}\mu_1(w_1v_1)\wedge\sigma_2(u_2).$$

$$\therefore d_{G_1\times G_2}((u_1,u_2)(v_1,u_2)) = \sum_{w_2\in V_2}\sigma_1(u_1)\wedge\mu_2(u_2w_2) + \sum_{w_1\in V_1, w_1\neq v_1}\mu_1(u_1w_1)\wedge\sigma_2(u_2) + \sum_{w_2\in V_2}\sigma_1(v_1)\wedge\mu_2(w_2u_2) + \sum_{w_1\in V_1, w_1\neq u_1}\mu_1(w_1v_1)\wedge\sigma_2(u_2) \quad (5.2)$$

In the following theorems, we find the degree of $((u_1,u_2)(u_1,v_2))$ and $((u_1,u_2)(v_1,u_2))$ in $G_1\times G_2$ in terms of those in G_1 and G_2 in some particular cases.

Theorem

Let $G_1:(\sigma_1,\mu_1)$ and $G_2:(\sigma_2,\mu_2)$ be two fuzzy graphs. If $\sigma_1 \geq \mu_2$ and $\sigma_2 \geq \mu_1$, then

- $d_{G_1\times G_2}((u_1,u_2)(u_1,v_2)) = 2d_{G_1}(u_1) + d_{G_2}(u_2v_2)$, if $(u_1,u_2)(u_1,v_2)\in E$,
- $d_{G_1\times G_2}((u_1,u_2)(v_1,u_2)) = d_{G_1}(u_1v_1) + 2d_{G_2}(u_2)$, if $(u_1,u_2)(v_1,u_2)\in E$.

Proof

We have, $\sigma_1 \geq \mu_2$ and $\sigma_2 \geq \mu_1$.

From (5.1), for any $(u_1,u_2)(u_1,v_2)\in E$,

$$d_{G_1\times G_2}((u_1,u_2)(u_1,v_2)) = \sum_{w_2\in V_2, w_2\neq v_2}\sigma_1(u_1)\wedge\mu_2(u_2w_2) + \sum_{w_1\in V_1}\mu_1(u_1w_1)\wedge\sigma_2(u_2)$$

$$+ \sum_{w_2 \in V_2, w_2 \neq u_2} \sigma_1(u_1) \wedge \mu_2(w_2 v_2) + \sum_{w_1 \in V_1} \mu_1(w_1 u_1) \wedge \sigma_2(v_2).$$

$$= \sum_{w_2 \in V_2, w_2 \neq v_2} \mu_2(u_2 w_2) + \sum_{w_1 \in V_1} \mu_1(u_1 w_1) + \sum_{w_2 \in V_2, w_2 \neq u_2} \mu_2(w_2 v_2) + \sum_{w_1 \in V_1} \mu_1(w_1 u_1)$$

$$= 2d_{G_1}(u_1) + d_{G_2}(u_2 v_2).$$

From (5.2), for any $(u_1, u_2)(v_1, u_2) \in E$,

$$d_{G_1 \times G_2}((u_1, u_2)(v_1, u_2)) = \sum_{w_2 \in V_2} \sigma_1(u_1) \wedge \mu_2(u_2 w_2) + \sum_{w_1 \in V_1, w_1 \neq v_1} \mu_1(u_1 w_1) \wedge \sigma_2(u_2)$$

$$+ \sum_{w_2 \in V_2} \sigma_1(v_1) \wedge \mu_2(w_2 u_2) + \sum_{w_1 \in V_1, w_1 \neq u_1} \mu_1(w_1 v_1) \wedge \sigma_2(u_2)$$

$$= \sum_{w_2 \in V_2} \mu_2(u_2 w_2) + \sum_{w_1 \in V_1, w_1 \neq v_1} \mu_1(u_1 w_1) + \sum_{w_2 \in V_2} \mu_2(w_2 u_2) + \sum_{w_1 \in V_1, w_1 \neq u_1} \mu_1(w_1 v_1)$$

$$= d_{G_1}(u_1 v_1) + 2d_{G_2}(u_2).$$

Theorem

Let $G_1 : (\sigma_1, \mu_1)$ and $G_2 : (\sigma_2, \mu_2)$ be two fuzzy graphs.

- If $\sigma_1 \leq \mu_2$ and σ_1 is a constant function with $\sigma_1(u) = c_1$ for all $u \in V_1$, then
  - For any $(u_1, u_2)(u_1, v_2) \in E$, $d_{G_1 \times G_2}((u_1, u_2)(u_1, v_2)) = 2d_{G_1}(u_1) + c_1(d_{G_2^*}(u_2) + d_{G_2^*}(v_2) - 2)$.
  - For any $(u_1, u_2)(v_1, u_2) \in E$, $d_{G_1 \times G_2}((u_1, u_2)(v_1, u_2)) = d_{G_1}(u_1 v_1) + 2c_1 d_{G_2^*}(u_2)$.
- If $\sigma_2 \leq \mu_1$ and σ_2 is a constant function with $\sigma_2(u) = c_2$ for all $u \in V_2$, then
  - For any $(u_1, u_2)(u_1, v_2) \in E$, $d_{G_1 \times G_2}((u_1, u_2)(u_1, v_2)) = 2c_2 d_{G_1^*}(u_1) + d_{G_2}(u_2 v_2)$.
  - For any $(u_1, u_2)(v_1, u_2) \in E$, $d_{G_1 \times G_2}((u_1, u_2)(v_1, u_2)) = c_2(d_{G_1^*}(u_1) + d_{G_1^*}(v_1) - 2) + 2d_{G_2}(u_2)$.

Proof

- We have $\sigma_1 \leq \mu_2$. By theorem*, $\sigma_2 \geq \mu_1$.
  - From (5.1), for any $(u_1, u_2)(u_1, v_2) \in E$,

$$d_{G_1 \times G_2}((u_1, u_2)(u_1, v_2)) = \sum_{w_2 \in V_2, w_2 \neq v_2} \sigma_1(u_1) \wedge \mu_2(u_2 w_2) + \sum_{w_1 \in V_1} \mu_1(u_1 w_1) \wedge \sigma_2(u_2) + \sum_{w_2 \in V_2, w_2 \neq u_2} \sigma_1(u_1) \wedge \mu_2(w_2 v_2)$$

$$+ \sum_{w_1 \in V_1} \mu_1(w_1 u_1) \wedge \sigma_2(v_2).$$

Since $\sigma_1(u) = c_1$ for all $u \in V_1$.

$$\therefore d_{G_1 \times G_2}((u_1,u_2)(u_1,v_2)) = \sum_{\substack{w_2 \in V_2, u_2w_2 \in E_2, \\ w_2 \neq v_2}} c_1 + \sum_{w_1 \in V_1} \mu_1(u_1w_1) + \sum_{\substack{w_2 \in V_2, w_2v_2 \in E_2, \\ w_2 \neq u_2}} c_1 + \sum_{w_1 \in V_1} \mu_1(w_1u_1)$$

$$= \sum_{u_2w_2 \in E_2, w_2 \neq v_2} c_1 + \sum_{w_1 \in V_1} \mu_1(u_1w_1) + \sum_{w_2v_2 \in E_2, w_2 \neq u_2} c_1 + \sum_{w_1 \in V_1} \mu_1(w_1u_1)$$

$$= c_1(d_{G_2^*}(u_2) - 1) + d_{G_1}(u_1) + c_1(d_{G_2^*}(v_2) - 1) + d_{G_1}(u_1)$$

$$= 2d_{G_1}(u_1) + c_1(d_{G_2^*}(u_2) + d_{G_2^*}(v_2) - 2).$$

- From (5.2), for any $(u_1,u_2)(v_1,u_2) \in E$,

$$d_{G_1 \times G_2}((u_1,u_2)(v_1,u_2)) = \sum_{w_2 \in V_2} \sigma_1(u_1) \wedge \mu_2(u_2w_2) + \sum_{w_1 \in V_1, w_1 \neq v_1} \mu_1(u_1w_1) \wedge \sigma_2(u_2) + \sum_{w_2 \in V_2} \sigma_1(v_1) \wedge \mu_2(w_2u_2)$$

$$+ \sum_{w_1 \in V_1, w_1 \neq u_1} \mu_1(w_1v_1) \wedge \sigma_2(u_2)$$

$$= \sum_{w_2 \in V_2, u_2w_2 \in E_2} \sigma_1(u_1) + \sum_{w_1 \in V_1, w_1 \neq v_1} \mu_1(u_1w_1) + \sum_{w_2 \in V_2, w_2u_2 \in E_2} \sigma_1(v_1) + \sum_{w_1 \in V_1, w_1 \neq u_1} \mu_1(w_1v_1)$$

$$= \sum_{u_2w_2 \in E_2} c_1 + d_{G_1}(u_1v_1) + \sum_{w_2u_2 \in E_2} c_1$$

$$= d_{G_1}(u_1v_1) + 2c_1 d_{G_2^*}(u_2) .$$

- Proof is similar to the proof of (1).

Remark

Let $G:(\sigma,\mu)$ be a fuzzy graph of Cartesian product of two given fuzzy graphs $G_1:(\sigma_1,\mu_1)$ and $G_2:(\sigma_2,\mu_2)$. Then (1) $\sigma = c_1$ if $\sigma_1 \leq \mu_2$ with

- $\sigma = c_2$ if $\sigma_2 \leq \mu_1$ with $\sigma_2 = c_2$.

Proof

By definition, $\sigma = \sigma_1 \wedge \sigma_2$.

Let $\sigma_1 \leq \mu_2$ with $\sigma_1 = c_1$.

Then $\sigma_1 \leq \min \mu_2 \leq \sigma_2$, (by definition fuzzy graph, $\min \mu_i \leq \sigma_i$ and $\mu_i \leq \max \sigma_i$).

$\Rightarrow \sigma_1 \leq \sigma_2$

$\Rightarrow \sigma_1 \wedge \sigma_2 = \sigma_1 = c_1$

$\Rightarrow \sigma = c_1$.

Similarly, if $\sigma_2 \le \mu_1$ with $\sigma_2 = c_2$, then $\sigma = c_2$.

DEGREE OF AN EDGE IN COMPOSITION

By definition, for any $(u_1, u_2) \in V_1 \times V_2$ and $((u_1, u_2)(v_1, v_2)) \in E$,

$$d_{G_1 \circ G_2}((u_1,u_2)(v_1,v_2)) = \sum_{\substack{(u_1,u_2)(w_1,w_2)\in E,\\ (w_1,w_2)\neq(v_1,v_2)}} (\mu_1 \circ \mu_2)((u_1,u_2)(w_1,w_2)) + \sum_{\substack{(w_1,w_2)(v_1,v_2)\in E,\\ (w_1,w_2)\neq(u_1,u_2)}} (\mu_1 \circ \mu_2)((w_1,w_2)(v_1,v_2)).$$

By using Cartesian product,

- If $u_1 = v_1, u_2 \neq v_2$, then

$$d_{G_1 \circ G_2}((u_1,u_2)(u_1,v_2)) = \sum_{\substack{(u_1,u_2)(w_1,w_2)\in E,\\ (w_1,w_2)\neq(u_1,v_2)}} (\mu_1 \circ \mu_2)((u_1,u_2)(w_1,w_2)) + \sum_{\substack{(w_1,w_2)(u_1,v_2)\in E,\\ (w_1,w_2)\neq(u_1,u_2)}} (\mu_1 \circ \mu_2)((w_1,w_2)(u_1,v_2))$$

$$d_{G_1 \circ G_2}((u_1,u_2)(u_1,v_2)) = \sum_{\substack{u_2w_2\in E_2, u_1=w_1,\\ w_2\neq v_2}} \sigma_1(u_1) \wedge \mu_2(u_2w_2) + \sum_{u_1w_1\in E_1, u_2=w_2} \mu_1(u_1w_1) \wedge \sigma_2(u_2)$$

$$+ \sum_{u_1w_1\in E_1, u_2\neq w_2} \mu_1(u_1w_1) \wedge \sigma_2(u_2) \wedge \sigma_2(w_2) + \sum_{\substack{w_2v_2\in E_2, w_1=u_1,\\ w_2\neq u_2}} \sigma_1(u_1) \wedge \mu_2(w_2v_2) + \sum_{w_1u_1\in E_1, w_2=v_2} \mu_1(w_1u_1) \wedge \sigma_2(v_2)$$

$$+ \sum_{w_1u_1\in E_1, w_2\neq v_2} \mu_1(w_1u_1) \wedge \sigma_2(w_2) \wedge \sigma_2(v_2) \quad (6.1)$$

- If $u_1 \neq v_1, u_2 = v_2$, then

$$d_{G_1 \circ G_2}((u_1,u_2)(v_1,u_2)) = \sum_{\substack{(u_1,u_2)(w_1,w_2)\in E,\\ (w_1,w_2)\neq(v_1,u_2)}} (\mu_1 \circ \mu_2)((u_1,u_2)(w_1,w_2)) + \sum_{\substack{(w_1,w_2)(v_1,u_2)\in E,\\ (w_1,w_2)\neq(u_1,u_2)}} (\mu_1 \circ \mu_2)((w_1,w_2)(v_1,u_2))$$

$$d_{G_1 \circ G_2}((u_1,u_2)(v_1,u_2)) = \sum_{u_2w_2\in E_2, u_1=w_1} \sigma_1(u_1) \wedge \mu_2(u_2w_2) + \sum_{\substack{u_1w_1\in E_1, u_2=w_2,\\ w_1\neq v_1}} \mu_1(u_1w_1) \wedge \sigma_2(u_2)$$

$$+ \sum_{u_1w_1\in E_1, u_2\neq w_2} \mu_1(u_1w_1) \wedge \sigma_2(u_2) \wedge \sigma_2(w_2) + \sum_{w_2u_2\in E_2, w_1=u_1} \sigma_1(v_1) \wedge \mu_2(w_2u_2) + \sum_{\substack{w_1v_1\in E_1, w_2=u_2,\\ w_1\neq u_1}} \mu_1(w_1v_1) \wedge \sigma_2(u_2)$$

$$+ \sum_{w_1v_1\in E_1, w_2\neq u_2} \mu_1(w_1v_1) \wedge \sigma_2(w_2) \wedge \sigma_2(u_2) \quad (6.2)$$

- If $u_1 \neq v_1, u_2 \neq v_2$, then

$$d_{G_1 \circ G_2}((u_1,u_2)(v_1,v_2)) = \sum_{\substack{(u_1,u_2)(w_1,w_2)\in E,\\ (w_1,w_2)\neq(v_1,u_2)}} (\mu_1 \circ \mu_2)((u_1,u_2)(w_1,w_2)) + \sum_{\substack{(w_1,w_2)(v_1,u_2)\in E,\\ (w_1,w_2)\neq(u_1,u_2)}} (\mu_1 \circ \mu_2)((w_1,w_2)(v_1,v_2))$$

$$d_{G_1 \circ G_2}((u_1,u_2)(v_1,v_2)) = \sum_{u_2w_2 \in E_2, u_1=w_1} \sigma_1(u_1) \wedge \mu_2(u_2w_2) + \sum_{u_1w_1 \in E_1, u_2=w_2} \mu_1(u_1w_1) \wedge \sigma_2(u_2)$$

$$+ \sum_{\substack{u_1w_1 \in E_1, u_2 \neq w_2, \\ w_1 \neq v_1 (or) w_2 \neq v_2}} \mu_1(u_1w_1) \wedge \sigma_2(u_2) \wedge \sigma_2(w_2) + \sum_{w_2v_2 \in E_2, w_1=u_1} \sigma_1(v_1) \wedge \mu_2(w_2v_2) + \sum_{w_1v_1 \in E_1, w_2=v_2} \mu_1(w_1v_1) \wedge \sigma_2(v_2)$$

$$+ \sum_{\substack{w_1v_1 \in E_1, w_2 \neq v_2, \\ w_1 \neq u_1 (or) w_2 \neq u_2}} \mu_1(w_1v_1) \wedge \sigma_2(w_2) \wedge \sigma_2(v_2) \quad (6.3)$$

Theorem

Let $G_1 : (\sigma_1, \mu_1)$ and $G_2 : (\sigma_2, \mu_2)$ be two fuzzy graphs. If $\sigma_1 \geq \mu_2$ and $\sigma_2 \geq \mu_1$, then for any $(u_1,u_2)(v_1,v_2) \in E$,

- $d_{G_1[G_2]}((u_1,u_2)(u_1,v_2)) = 2p_2 d_{G_1}(u_1) + d_{G_2}(u_2v_2)$,
- $d_{G_1[G_2]}((u_1,u_2)(v_1,u_2)) = d_{G_1}(u_1v_1) + 2d_{G_2}(u_2) + (p_2 - 1)(d_{G_1}(u_1) + d_{G_1}(v_1))$,
- $d_{G_1[G_2]}((u_1,u_2)(v_1,v_2)) = d_{G_1}(u_1v_1) + (p_2 - 1)(d_{G_1}(u_1) + d_{G_1}(v_1)) + d_{G_2}(u_2) + d_{G_2}(v_2)$.

Proof

We have $\sigma_1 \geq \mu_2$ and $\sigma_2 \geq \mu_1$.

- From (6.1), for any $(u_1,u_2)(u_1,v_2) \in E$,

$$d_{G_1 \circ G_2}((u_1,u_2)(u_1,v_2)) = \sum_{\substack{u_2w_2 \in E_2, u_1=w_1, \\ w_2 \neq v_2}} \sigma_1(u_1) \wedge \mu_2(u_2w_2) + \sum_{u_1w_1 \in E_1, u_2=w_2} \mu_1(u_1w_1) \wedge \sigma_2(u_2)$$

$$+ \sum_{u_1w_1 \in E_1, u_2 \neq w_2} \mu_1(u_1w_1) \wedge \sigma_2(u_2) \wedge \sigma_2(w_2) + \sum_{\substack{w_2v_2 \in E_2, w_1=u_1, \\ w_2 \neq u_2}} \sigma_1(u_1) \wedge \mu_2(w_2v_2) + \sum_{w_1u_1 \in E_1, w_2=v_2} \mu_1(w_1u_1) \wedge \sigma_2(v_2)$$

$$+ \sum_{w_1u_1 \in E_1, w_2 \neq v_2} \mu_1(w_1u_1) \wedge \sigma_2(w_2) \wedge \sigma_2(v_2)$$

$$= \sum_{u_2w_2 \in E_2, w_2 \neq v_2} \mu_2(u_2w_2) + \sum_{u_1w_1 \in E_1} \mu_1(u_1w_1) + \sum_{u_1w_1 \in E_1, u_2 \neq w_2} \mu_1(u_1w_1) + \sum_{w_2v_2 \in E_2, w_2 \neq u_2} \mu_2(w_2v_2) + \sum_{w_1u_1 \in E_1} \mu_1(w_1u_1) + \sum_{w_1u_1 \in E_1, w_2 \neq v_2} \mu_1(w_1u_1)$$

$$= \sum_{w_2 \in V_2, w_2 \neq v_2} \mu_2(u_2w_2) + \sum_{w_1 \in V_1} \mu_1(u_1w_1) + |V_2 - \{u_2\}| \sum_{w_1 \in V_1} \mu_1(u_1w_1) + \sum_{w_2 \in V_2, w_2 \neq u_2} \mu_2(w_2v_2) + \sum_{w_1 \in V_1} \mu_1(w_1u_1)$$

$$+ |V - \{v_2\}| \sum_{w_1 \in V_1} \mu_1(w_1u_1)$$

$$= \sum_{w_2 \in V_2, w_2 \neq v_2} \mu_2(u_2w_2) + \sum_{w_2 \in V_2, w_2 \neq u_2} \mu_2(w_2v_2) + d_{G_1}(u_1) + (p_2 - 1)d_{G_1}(u_1) + d_{G_1}(u_1) + (p_2 - 1)d_{G_1}(u_1)$$

$$\therefore d_{G_1[G_2]}((u_1,u_2)(u_1,v_2)) = d_{G_2}(u_2v_2) + 2p_2 d_{G_1}(u_1).$$

- From (6.2), for any $(u_1,u_2)(v_1,u_2) \in E$,

$$d_{G_1 \circ G_2}((u_1,u_2)(v_1,u_2)) = \sum_{u_2w_2 \in E_2, u_1=w_1} \sigma_1(u_1) \wedge \mu_2(u_2w_2) + \sum_{\substack{u_1w_1 \in E_1, u_2=w_2, \\ w_1 \neq v_1}} \mu_1(u_1w_1) \wedge \sigma_2(u_2)$$

$$+ \sum_{u_1w_1 \in E_1, u_2 \neq w_2} \mu_1(u_1w_1) \wedge \sigma_2(u_2) \wedge \sigma_2(w_2) + \sum_{w_2u_2 \in E_2, w_1=u_1} \sigma_1(v_1) \wedge \mu_2(w_2u_2) + \sum_{\substack{w_1v_1 \in E_1, w_2=u_2, \\ w_1 \neq u_1}} \mu_1(w_1v_1) \wedge \sigma_2(u_2)$$

$$+ \sum_{w_1v_1 \in E_1, w_2 \neq u_2} \mu_1(w_1v_1) \wedge \sigma_2(w_2) \wedge \sigma_2(u_2)$$

$$= \sum_{w_2 \in V_2} \mu_2(u_2w_2) + \sum_{w_1 \in V_1, w_1 \neq v_1} \mu_1(u_1w_1) + |V_2 - \{u_2\}| \sum_{w_1 \in V_1} \mu_1(u_1w_1) + \sum_{w_2 \in V_2} \mu_2(w_2u_2) + \sum_{w_1 \in V_1, w_1 \neq u_1} \mu_1(w_1v_1)$$

$$+ |V_2 - \{u_2\}| \sum_{w_1 \in V_1} \mu_1(w_1v_1)$$

$$= d_{G_2}(u_2) + d_{G_1}(u_1v_1) + (p_2 - 1)(d_{G_1}(u_1)) + d_{G_2}(u_2) + (p_2 - 1)(d_{G_1}(v_1))$$

$$\therefore d_{G_1[G_2]}((u_1,u_2)(v_1,u_2)) = d_{G_1}(u_1v_1) + 2d_{G_2}(u_2) + (p_2 - 1)(d_{G_1}(u_1) + d_{G_1}(v_1)).$$

- From (6.3), for any $(u_1,u_2)(v_1,v_2) \in E$,

$$d_{G_1 \circ G_2}((u_1,u_2)(v_1,v_2)) = \sum_{u_2w_2 \in E_2, u_1=w_1} \sigma_1(u_1) \wedge \mu_2(u_2w_2) + \sum_{u_1w_1 \in E_1, u_2=w_2} \mu_1(u_1w_1) \wedge \sigma_2(u_2)$$

$$+ \sum_{\substack{u_1w_1 \in E_1, u_2 \neq w_2, \\ w_1 \neq v_1 (or) w_2 \neq v_2}} \mu_1(u_1w_1) \wedge \sigma_2(u_2) \wedge \sigma_2(w_2) + \sum_{w_2v_2 \in E_2, w_1=u_1} \sigma_1(v_1) \wedge \mu_2(w_2v_2) + \sum_{w_1v_1 \in E_1, w_2=v_2} \mu_1(w_1v_1) \wedge \sigma_2(v_2)$$

$$+ \sum_{\substack{w_1v_1 \in E_1, w_2 \neq v_2, \\ w_1 \neq u_1 (or) w_2 \neq u_2}} \mu_1(w_1v_1) \wedge \sigma_2(w_2) \wedge \sigma_2(v_2)$$

$$= \sum_{w_2 \in V_2} \mu_2(u_2w_2) + \sum_{w_1 \in V_1} \mu_1(u_1w_1) + \sum_{u_1w_1 \in E_1, u_2 \neq w_2} \mu_1(u_1w_1) \wedge \sigma_2(u_2) \wedge \sigma_2(w_2) - \mu_1(u_1v_1) + \sum_{w_2 \in V_2} \mu_2(w_2v_2)$$

$$+ \sum_{w_1 \in V_1} \mu_1(w_1v_1) + \sum_{w_1v_1 \in E_1, w_2 \neq v_2} \mu_1(w_1v_1) \wedge \sigma_2(w_2) \wedge \sigma_2(v_2) - \mu_1(u_1v_1)$$

$$= d_{G_2}(u_2) + d_{G_1}(u_1) + |V_2 - \{u_2\}| \sum_{w_1 \in V_1} \mu_1(u_1w_1) - \mu_1(u_1v_1) + d_{G_2}(v_2) + d_{G_1}(v_1) +$$

$$|V_2 - \{v_2\}| \sum_{w_1 \in V_1} \mu_1(w_1v_1) - \mu_1(u_1v_1)$$

$$= d_{G_2}(u_2) + d_{G_1}(u_1) + d_{G_1}(v_1) - 2\mu_1(u_1v_1) + (p_2 - 1)(d_{G_1}(u_1)) + d_{G_2}(v_2) + (p_2 - 1)(d_{G_1}(v_1))$$

$$\therefore d_{G_1[G_2]}((u_1,u_2)(v_1,v_2)) = d_{G_1}(u_1v_1) + (p_2 - 1)(d_{G_1}(u_1) + d_{G_1}(v_1)) + d_{G_2}(u_2) + d_{G_2}(v_2).$$

Theorem

Let $G_1:(\sigma_1,\mu_1)$ and $G_2:(\sigma_2,\mu_2)$ be two fuzzy graphs.

- If $\sigma_1 \le \mu_2$ and σ_1 is a constant function with $\sigma_1(u)=c_1$ for all $u\in V_1$, then for any $(u_1,u_2)(v_1,v_2)\in E$,
  - $d_{G_1[G_2]}((u_1,u_2)(u_1,v_2)) = 2p_2 d_{G_1}(u_1) + c_1(d_{G_2^*}(u_2)+d_{G_2^*}(v_2)-2)$,
  - $d_{G_1[G_2]}((u_1,u_2)(v_1,u_2)) = d_{G_1}(u_1v_1) + (p_2-1)(d_{G_1}(u_1)+d_{G_1}(v_1)) + 2c_1 d_{G_2^*}(u_2)$,
  - $d_{G_1[G_2]}((u_1,u_2)(v_1,v_2)) = d_{G_1}(u_1v_1) + (p_2-1)(d_{G_1}(u_1)+d_{G_1}(v_1)) + c_1(d_{G_2^*}(u_2)+d_{G_2^*}(v_2))$.
- If $\sigma_2 \le \mu_1$ and σ_2 is a constant function with $\sigma_2(u)=c_2$ for all $u\in V_2$, then for any $(u_1,u_2)(v_1,v_2)\in E$,
  - $d_{G_1[G_2]}((u_1,u_2)(u_1,v_2)) = 2c_2 p_2 d_{G_1^*}(u_1) + d_{G_2}(u_2v_2)$,
  - $d_{G_1[G_2]}((u_1,u_2)(v_1,u_2)) = c_2(p_2(d_{G_1^*}(u_1)+d_{G_1^*}(v_1))-2) + 2d_{G_2}(u_2)$,
  - $d_{G_1[G_2]}((u_1,u_2)(v_1,v_2)) = c_2(p_2(d_{G_1^*}(u_1)+d_{G_1^*}(v_1))-2) + d_{G_2}(u_2) + d_{G_2}(v_2)$.

Proof:

- We have $\sigma_1 \le \mu_2$. Then by theorem*, $\sigma_2 \ge \mu_1$.

From (6.1), for any $(u_1,u_2)(u_1,v_2)\in E$, $d_{G_1\circ G_2}((u_1,u_2)(u_1,v_2))$

$$= \sum_{\substack{u_2w_2\in E_2, u_1=w_1,\\ w_2\ne v_2}} \sigma_1(u_1)\wedge\mu_2(u_2w_2) + \sum_{u_1w_1\in E_1, u_2=w_2} \mu_1(u_1w_1)\wedge\sigma_2(u_2) + \sum_{u_1w_1\in E_1, u_2\ne w_2} \mu_1(u_1w_1)\wedge\sigma_2(u_2)\wedge\sigma_2(w_2)$$

$$+ \sum_{\substack{w_2v_2\in E_2, w_1=u_1,\\ w_2\ne u_2}} \sigma_1(u_1)\wedge\mu_2(w_2v_2) + \sum_{w_1u_1\in E_1, w_2=v_2} \mu_1(w_1u_1)\wedge\sigma_2(v_2) + \sum_{w_1u_1\in E_1, w_2\ne v_2} \mu_1(w_1u_1)\wedge\sigma_2(w_2)\wedge\sigma_2(v_2)$$

$$= \sum_{u_2w_2\in E_2, w_2\ne v_2} \sigma_1(u_1) + \sum_{u_1w_1\in E_1} \mu_1(u_1w_1) + \sum_{u_1w_1\in E_1, u_2\ne w_2} \mu_1(u_1w_1) + \sum_{w_2v_2\in E_2, w_2\ne u_2} \sigma_1(u_1) + \sum_{w_1u_1\in E_1} \mu_1(w_1u_1) + \sum_{w_1u_1\in E_1, w_2\ne v_2} \mu_1(w_1u_1)$$

$$= c_1(d_{G_2^*}(u_2)-1) + d_{G_1}(u_1) + |V_2-\{u_2\}| \sum_{w_1\in V_1} \mu_1(u_1w_1) + c_1(d_{G_2^*}(v_2)-1) + d_{G_1}(u_1)$$

$$+ |V_2-\{v_2\}| \sum_{w_1\in V_1} \mu_1(w_1u_1)$$

$$= c_1(d_{G_2^*}(u_2)+d_{G_2^*}(v_2)-2) + d_{G_1}(u_1) + (p_2-1)d_{G_1}(u_1) + d_{G_1}(u_1) + (p_2-1)d_{G_1}(u_1)$$

$$\therefore d_{G_1[G_2]}((u_1,u_2)(u_1,v_2)) = 2p_2 d_{G_1}(u_1) + c_1(d_{G_2^*}(u_2)+d_{G_2^*}(v_2)-2).$$

From (6.2), for any $(u_1,u_2)(v_1,u_2)\in E$, $d_{G_1\circ G_2}((u_1,u_2)(v_1,u_2))$

$$= \sum_{u_2w_2\in E_2,u_1=w_1}\sigma_1(u_1)\wedge\mu_2(u_2w_2)+\sum_{\substack{u_1w_1\in E_1,u_2=w_2,\\ w_1\neq v_1}}\mu_1(u_1w_1)\wedge\sigma_2(u_2)+\sum_{u_1w_1\in E_1,u_2\neq w_2}\mu_1(u_1w_1)\wedge\sigma_2(u_2)\wedge\sigma_2(w_2)$$

$$+\sum_{w_2u_2\in E_2,w_1=u_1}\sigma_1(v_1)\wedge\mu_2(w_2u_2)+\sum_{\substack{w_1v_1\in E_1,w_2=u_2,\\ w_1\neq u_1}}\mu_1(w_1v_1)\wedge\sigma_2(u_2)+\sum_{w_1v_1\in E_1,w_2\neq u_2}\mu_1(w_1v_1)\wedge\sigma_2(w_2)\wedge\sigma_2(u_2)$$

$$= \sum_{u_2w_2\in E_{21}}\sigma_1(u_1)+\sum_{u_1w_1\in E_1,w_1\neq v_1}\mu_1(u_1w_1)+\sum_{u_1w_1\in E_1,u_2\neq w_2}\mu_1(u_1w_1)+\sum_{w_2u_2\in E_2}\sigma_1(v_1)+\sum_{w_1v_1\in E_1,w_1\neq u_1}\mu_1(w_1v_1)+\sum_{w_1v_1\in E_1,w_2\neq u_2}\mu_1(w_1v_1)$$

$$= c_1d_{G_2^*}(u_2)+\sum_{w_1\in V_1,w_1\neq v_1}\mu_1(u_1w_1)+\sum_{w_1\in V_1,w_1\neq u_1}\mu_1(w_1v_1)+\left|V_2-\{u_2\}\right|\sum_{w_1\in V_1}\mu_1(u_1w_1)+c_1d_{G_2^*}(u_2)+\left|V_2-\{u_2\}\right|\sum_{w_1\in V_1}\mu_1(w_1v_1)$$

$$=2c_1d_{G_2^*}(u_2)+d_{G_1}(u_1v_1)+(p_2-1)(d_{G_1}(u_1))+(p_2-1)(d_{G_1}(v_1))$$

$$\therefore d_{G_1[G_2]}((u_1,u_2)(v_1,u_2))=d_{G_1}(u_1v_1)+(p_2-1)(d_{G_1}(u_1)+d_{G_1}(v_1))+2c_1d_{G_2^*}(u_2).$$

From (6.3), for any $(u_1,u_2)(v_1,v_2)\in E$, $d_{G_1\circ G_2}((u_1,u_2)(v_1,v_2))$

$$= \sum_{u_2w_2\in E_2,u_1=w_1}\sigma_1(u_1)\wedge\mu_2(u_2w_2)+\sum_{u_1w_1\in E_1,u_2=w_2}\mu_1(u_1w_1)\wedge\sigma_2(u_2)+\sum_{\substack{u_1w_1\in E_1,u_2\neq w_2,\\ w_1\neq v_1(or)w_2\neq v_2}}\mu_1(u_1w_1)\wedge\sigma_2(u_2)\wedge\sigma_2(w_2)$$

$$+\sum_{w_2v_2\in E_2,w_1=u_1}\sigma_1(v_1)\wedge\mu_2(w_2v_2)+\sum_{w_1v_1\in E_1,w_2=v_2}\mu_1(w_1v_1)\wedge\sigma_2(v_2)+\sum_{\substack{w_1v_1\in E_1,w_2\neq v_2,\\ w_1\neq u_1(or)w_2\neq u_2}}\mu_1(w_1v_1)\wedge\sigma_2(w_2)\wedge\sigma_2(v_2)$$

$$= \sum_{u_2w_2\in E_2}\sigma_1(u_1)+\sum_{u_1w_1\in E_1}\mu_1(u_1w_1)+\sum_{u_1w_1\in E_1,u_2\neq w_2}\mu_1(u_1w_1)-\mu_1(u_1v_1)+\sum_{w_2v_2\in E_2}\sigma_1(v_1)+\sum_{w_1v_1\in E_1}\mu_1(w_1v_1)$$

$$+\sum_{w_1v_1\in E_1,w_2\neq v_2}\mu_1(w_1v_1)-\mu_1(u_1v_1)$$

$$= c_1d_{G_2^*}(u_2)+d_{G_1}(u_1)+\left|V_2-\{u_2\}\right|d_{G_1}(u_1)-2\mu_1(u_1v_1)+c_1d_{G_2^*}(v_2)+d_{G_1}(v_1)+\left|V_2-\{v_2\}\right|d_{G_1}(v_1)$$

$$= c_1(d_{G_2^*}(u_2)+d_{G_2^*}(v_2))+d_{G_1}(u_1)+d_{G_1}(v_1)-2\mu_1(u_1v_1)+(p_2-1)d_{G_1}(u_1)+(p_2-1)d_{G_1}(v_1)$$

$$\therefore d_{G_1[G_2]}((u_1,u_2)(v_1,v_2))=d_{G_1}(u_1v_1)+(p_2-1)(d_{G_1}(u_1)+d_{G_1}(v_1))+c_1(d_{G_2^*}(u_2)+d_{G_2^*}(v_2)).$$

- We have $\sigma_2\le\mu_1$. Then by theorem*, $\sigma_1\ge\mu_2$.

From (6.1), for any $(u_1,u_2)(u_1,v_2)\in E$, $d_{G_1\circ G_2}((u_1,u_2)(u_1,v_2))$

$$= \sum_{u_2w_2\in E_2,w_2\neq v_2}\mu_2(u_2w_2)+\sum_{u_1w_1\in E_1}\sigma_2(u_2)+\sum_{u_1w_1\in E_1,u_2\neq w_2}\sigma_2(u_2)\wedge\sigma_2(w_2)+\sum_{w_2v_2\in E_2,w_2\neq u_2}\mu_2(w_2v_2)+\sum_{w_1u_1\in E_1}\sigma_2(v_2)$$

$$+ \sum_{w_1u_1 \in E_1, w_2 \neq v_2} \sigma_2(w_2) \wedge \sigma_2(v_2)$$

$$= \sum_{u_2w_2 \in E_2, w_2 \neq v_2} \mu_2(u_2w_2) + \sum_{w_2v_2 \in E_2, w_2 \neq u_2} \mu_2(w_2v_2) + c_2 d_{G_1^*}(u_1) + |V_2 - \{u_2\}| \sum_{u_1w_1 \in E_1} c_2 + c_2 d_{G_1^*}(u_1) + |V_2 - \{v_2\}| \sum_{w_1u_1 \in E_1} c_2$$

$$= d_{G_2}(u_2v_2) + 2c_2 d_{G_1^*}(u_1) + (p_2 - 1)c_2 d_{G_1^*}(u_1) + (p_2 - 1)c_2 d_{G_1^*}(u_1)$$

$$\therefore d_{G_1[G_2]}((u_1, u_2)(u_1, v_2)) = 2c_2 p_2 d_{G_1^*}(u_1) + d_{G_2}(u_2v_2).$$

From (6.2), for any $(u_1, u_2)(v_1, u_2) \in E$, $d_{G_1 \circ G_2}((u_1, u_2)(v_1, u_2))$

$$= \sum_{u_2w_2 \in E_2} \mu_2(u_2w_2) + \sum_{u_1w_1 \in E_1, w_1 \neq v_1} \sigma_2(u_2) + \sum_{u_1w_1 \in E_1, u_2 \neq w_2} \sigma_2(u_2) \wedge \sigma_2(w_2) + \sum_{w_2u_2 \in E_2} \mu_2(w_2u_2) + \sum_{w_1v_1 \in E_1, w_1 \neq u_1} \sigma_2(u_2)$$

$$+ \sum_{w_1v_1 \in E_1, w_2 \neq u_2} \sigma_2(w_2) \wedge \sigma_2(u_2)$$

$$= d_{G_2}(u_2) + c_2(d_{G_1^*}(u_1) - 1) + (p_2 - 1)c_2 d_{G_1^*}(u_1) + d_{G_2}(u_2) + c_2(d_{G_1^*}(v_1) - 1) + (p_2 - 1)c_2 d_{G_1^*}(v_1)$$

$$\therefore d_{G_1[G_2]}((u_1, u_2)(v_1, u_2)) = c_2(p_2(d_{G_1^*}(u_1) + d_{G_1^*}(v_1)) - 2) + 2d_{G_2}(u_2).$$

From (6.3), for any $(u_1, u_2)(v_1, v_2) \in E$, $d_{G_1 \circ G_2}((u_1, u_2)(v_1, v_2))$

$$= \sum_{u_2w_2 \in E_2} \mu_2(u_2w_2) + \sum_{u_1w_1 \in E_1} \sigma_2(u_2) + \sum_{\substack{u_1w_1 \in E_1, u_2 \neq w_2, \\ w_1 \neq v_1 (or) w_2 \neq v_2}} \sigma_2(u_2) \wedge \sigma_2(w_2) + \sum_{w_2v_2 \in E_2} \mu_2(w_2v_2) + \sum_{w_1v_1 \in E_1} \sigma_2(v_2) +$$

$$\sum_{\substack{w_1v_1 \in E_1, w_2 \neq v_2, \\ w_1 \neq u_1 (or) w_2 \neq u_2}} \sigma_2(w_2) \wedge \sigma_2(v_2)$$

$$= d_{G_2}(u_2) + c_2 d_{G_1^*}(u_1) + c_2((p_2 - 1)d_{G_1^*}(u_1) - 1) + d_{G_2}(v_2) + c_2 d_{G_1^*}(v_1) + c_2((p_2 - 1)d_{G_1^*}(v_1) - 1)$$

$$\therefore d_{G_1[G_2]}((u_1, u_2)(v_1, v_2)) = c_2(p_2(d_{G_1^*}(u_1) + d_{G_1^*}(v_1)) - 2) + d_{G_2}(u_2) + d_{G_2}(v_2).$$

Remark

When both σ = 1 and μ = 1, all the above formulae coincide with the corresponding formulae of crisp graphs.

CONCLUSIONS

In this paper, we have found the degree of edges in $G_1 \times G_2$ and $G_1[G_2]$ in terms of the degree of vertices and edges in G_1 and G_2 and also in terms of the degree of vertices in G_1* and G_2* under some conditions.

They will we more helpful especially when the graphs are very large. Also they will be useful in studying various conditions, properties of Cartesian product and composition of two fuzzy graphs and also used to further study for edge regular on some fuzzy graphs.

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